Calculation Policy

Manford Primary School

LONDON BOROUGH OF REDBRIDGE
MANFORD
PRIMARY SCHOOL

Believe in Yourself

INTRODUCTION;

This calculation policy, taken from the new National Curriculum for Mathematics, has been written in line with the programmes of study. It provides guidance on appropriate calculation methods and progression. The content is set out in stages under the following headings: addition, subtraction, multiplication and division.

AIMS OF THE POLICY:

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop efficient, reliable and formal written methods of calculation for all operations
- To ensure that children can use these methods fluently with confidence and understanding

HOW TO USE THIS POLICY:

Under the 2014 National Curriculum, *'pupils should be fluent in written methods for all four operations'*, including applying these skills with more than 4 digit numbers. Therefore, we are working towards ALL children having a grasp of these methods, with a conceptual understanding of how the method works and what the stages mean.

- Use the policy as the basis of your planning but ensure you use previous or following years' guidance to allow for personalised learning (Learning without Limits).
- Always use AfL to identify suitable next steps in calculation for groups of children.
- If, at any time, children are making significant errors, return to the previous stage in calculation.
- Always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate. The list of resources is not exhaustive and the policy doesn't recommend using one set of resources - rather can be supplemented with a variety of resources, which work for your class.
- For each of the four arithmetic rules, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept; children should master all three phases within a year group's scheme of work.
- Generalisations and key questions at the end of each stage supports teacher assessment.
- Expectations for each year group are listed in bold at the end of each section.

This policy is purposely set out as a progression of mathematical skill. The focus must always remain on the breadth and depth of mathematics, rather than accelerating through concepts, in order to deepen their conceptual understanding by tackling challenging varied problems.

<table>
<tr><th colspan="3">ADDITION GUIDELINES – Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, ‘is equal to’ ‘is’</th></tr>
<tr><th>Stage One</th><th>Stage Two</th><th>Stage Three</th></tr>
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<td>

<u>Prerequisite skills</u> (based on the practical)
Counting numbers to 20

(using familiar / practical resources)

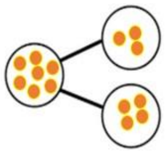
Place numbers to 20 in order

Teaching concrete, pictorial and abstract alongside:
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears etc.)

<u>Concrete –</u>

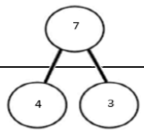
 

<u>Pictorial –</u>



<u>Abstract –</u>

4 + 3 = 7 (four is a part, 3 is a part and the whole is seven)



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<u>Prerequisite skills</u> (based on the practical)

Relate number bonds to 10 to add multiples of 10 up to a total of 100 e.g. if 3 + 4 is 7 then 30 + 40 is 70

Use familiar objects to recognise the place value of 2 digit numbers.

Recognise and explain 34 is ‘3 tens and 4 ones’

Progressing to: PARTITIONING AND RECOMBINING
<u>Partition into tens and ones and recombine</u>

15 + 23 = 10 + 5 + 20 + 3
= 30 + 8
= 38

Model this on a bead bar and practise on 100-beadstrings, showing the ‘collection’ of 10s and then the ones. i.e. “2 tens and 1 ten makes 3 tens, which is 30. Then 3 and 2 makes 5 ones. Altogether we can see 3 tens and 5 ones, which is 35.” Check by counting in tens and ones along the bead bar. Model and practise

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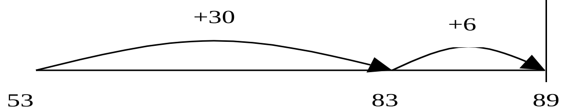
<u>Partition into tens and ones</u>

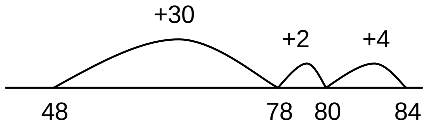
- Partition one number and recombine.
- Count on by partitioning the second number only e.g.

36 + 53 = 53 + 30 + 6
= 83 + 6
= 89

As modelled below as necessary



Children need to be secure adding multiples of 10 to any two-digit number including those that are not multiples of 10.
48 + 36 = 84



First J10 then T10

<u>Add a near multiple of 10 to a two-digit number (Overjumping – OJ)</u>
Secure mental methods by using a number line to model the method. Continue as in Stage 2 but with appropriate numbers

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ADDITION GUIDELINES – Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is'

Stage One

Bonds up to 10 and to make 10

1 more than a number

Addition as combining groups

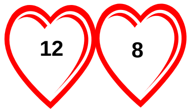
1, 2, 3, 4 1, 2, 3

1, 2, 3, 4, 5, 6, 7

Addition as counting on

Doubling numbers within 20

Number bonds to 20



+ / = signs and missing numbers

Stage Two

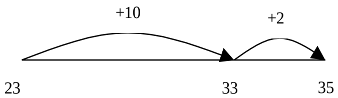
with place value arrow cards, Numicon, bead strings or Dienes, using known facts and place value to calculate each step.

Count on in tens and ones

J10 (jumping in 10s)

23 + 12 = 23 + 10 + 2
= 33 + 2
= 35

Model this on a number line starting at 23 and jumping 10 (J10) to make 33 and then add 2 in one jump.



The Empty Number Line:

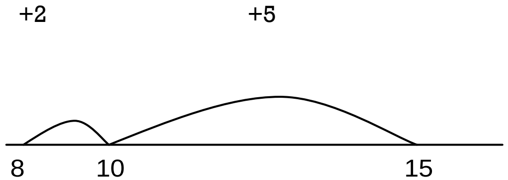
T10 (Targeting the 10, partitioning and bridging through 10)

The steps in addition often bridge through a multiple of 10

e.g.

Children should be able to partition the 7 to relate adding the 2 first to target the 10 and then the 5.

8 + 7 = 15



+ / = signs and missing numbers

Continue using a range of equations as in Stage 1 but with appropriate, larger numbers.

Extend to

Stage Three

E.g. 35 + 19 is the same as 35 + 20 – 1.

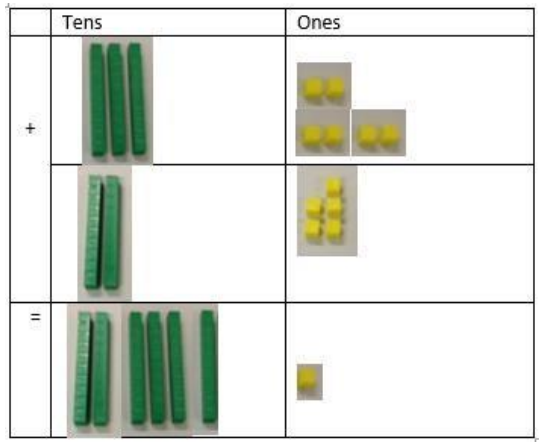
Once a child is able to add 3 digit numbers on a number line securely move on to vertical expansion.

+ / = signs and missing numbers

Continue using a range of equations as in Stage 1 and 2 but with appropriate, larger numbers.

Concrete –

TO + TO using base 10. Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. 36 + 25



ADDITION GUIDELINES – Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is'

Stage One

Children need to understand the concept of equality before using the '=' sign. **Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.**

2 = 1+ 1

2 + 3 = 4 + 1   3 = 3   2 + 2 + 2 = 4 + 2

Missing numbers need to be placed in all possible places.

3 + 4 = □	□ = 3 + 4
3 + □ = 7	7 = □ + 4
□ + 4 = 7	7 = 3 + □
□ + ∇ = 7	7 = □ + ∇

Expectations in Yr 1

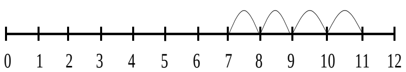
Children to develop an understanding of equality e.g

6 + □ = 11 and 6 + 5 = 5 + □   6 + 5 = □ + 4

The Number Line

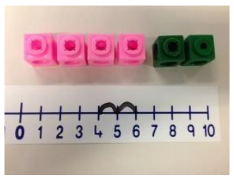
Children use a numbered line to count on in ones. Children use number lines and practical resources to support calculation and teachers *model* the use of the number line.

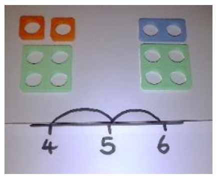
e.g. 7+ 4:



Concrete –

Counting on using number lines by using cubes or Numicon.





Stage Two

24 + 5 = 20 + □

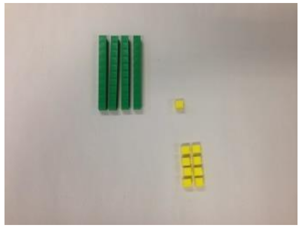
and

32 + □ + □ = 100   35 = 1 + □ + 5

Concrete –

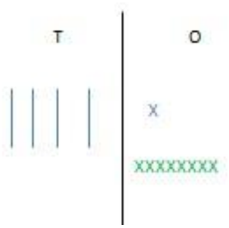
***TO + O* using base 10.** Continue to develop understanding of partitioning and place value.

e.g - 41 + 8



Pictorial –

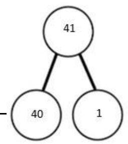
Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones.



Expectations in Yr 2

Concrete–

41 + 8



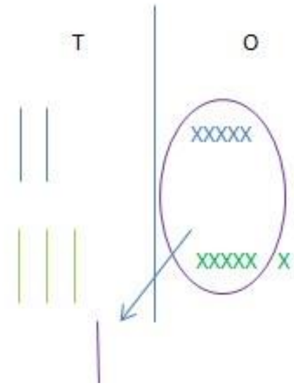
1 + 8 = 9

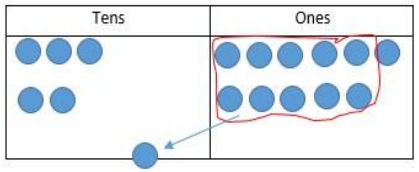
40 + 9 = 49

Stage Three

Pictorial –

This could be done one of two ways:





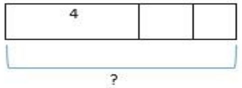
Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.

ADDITION GUIDELINES – Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, ‘is equal to’ ‘is’

Stage One

Pictorial – A bar model which encourages the children to count on.



The abstract number line -

What is 2 more than 4? What is the sum of 4 and 2? What’s the total of 4 and 2? 4 + 2

Number line Teaching Points:

Always work with numbers reading from left to right (smallest to largest), whatever the operation of the calculation.
Numbers (‘landmarks’) are written below the line.
Size of the ‘jumps’ are written above the ‘jumps’.

Generalisations

- True or false? Addition makes numbers bigger.
- True or false? You can add numbers in any order and still get the same answer.

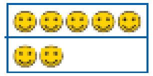
Some Key Questions

How many altogether? How many more to make…? I add …more. What is the total? How many more is… than…? How much more is…? One more, two more, ten more…
What can you see here?
Is this true or false?
What is the same? What is different?

Stage Two

Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



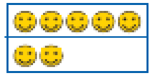
7 + ? = 10

Some Key Questions

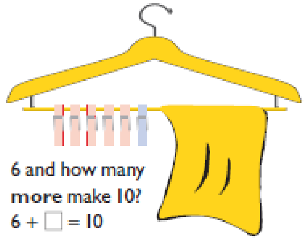
How many altogether? How many more to make…? How many more is… than…? How much more is…?
Is this true or false?

If I know that 17 + 2 = 19, what else do I know? (e.g. 2 + 17 = 19; 19 – 17 = 2; 19 – 2 = 17; 190 – 20 = 170 etc).
What do you notice? What patterns can you see?

Stage Three



7 + ? = 10



Some Key Questions

How many altogether? How many more to make…? How many more is… than…? How much more is…?
Is this true or false?

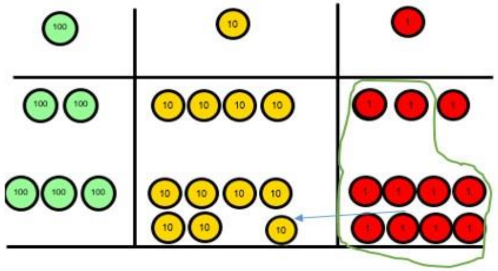
If I know that 17 + 2 = 19, what else do I know? (e.g. 2 + 17 = 19; 19 – 17 = 2; 19 – 2 = 17; 190 – 20 = 170 etc).
What do you notice? What patterns can you see?

ADDITION GUIDELINES

Stage Four

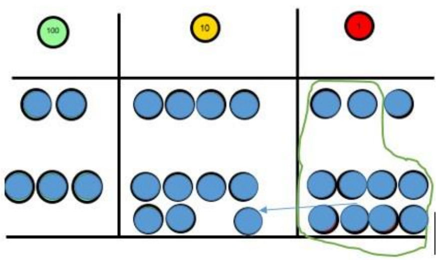
Concrete –

Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract

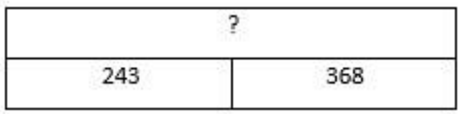


Pictorial –

Children to represent the counters e.g. like the image below



If the children are completing a word problem, draw a bar model to represent what it's asking them to do



Number line –

Stage Five

It is crucial to know or be able to derive key number facts TO + TO mentally or with jottings before progressing to Stage Five.

Adding the least significant digits first

247
+ 176
13 (7+6)
110 (40 + 70)
300 (200 + 100)
423

Expectations in Yr 3

Working from left to right:
'Read' the answer from left to right, using knowledge of place value and referring to the value of each digit i.e.:"four hundred and twenty three" NOT adding up columns for the final answer

Moving on to

247
+ 376 (without use of brackets)
13
110
500
623

Moving on to a compact method

247
+ 376
623
1 1

Expectations in Yr 3

Working from right to left: "7 + 6 is 13.

Stage Six

Extend to numbers with at least four digits

3587 + 675 = 4262

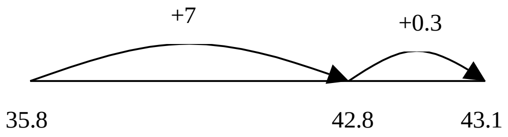
3587
+ 675
4262
1 1 1

Expectations in Yr 4, 5

Revert to expanded methods if the children experience any difficulty.

Partition into hundreds, tens, ones and decimal fractions and recombine

Either partition both numbers and recombine or partition the second number only e.g.
35.8 + 7.3 = 35.8 + 7 + 0.3
= 42.8 + 0.3
= 43.1



Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

72.8
+ 54.6
127.4
1 1

Expectations in Yr 5,6

Extend to numbers with more than 4 digits or

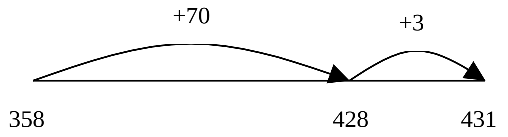
Partition into hundreds, tens and ones and recombine

Either partition both numbers and recombine or partition the second number only e.g.

358 + 73 = 358 + 70 + 3

= 428 + 3

= 431



Horizontal Expansion

367 + 185 = 552

```
 367
+185
 400  (300+100)
 140  (60+80)
  12   (7+5)
 552
```

Expectations in Yr 3

Moving on to

367 + 185 = 552

```
 367          (without use of brackets)
+185
 400
 140
  12
 552
```

Why most significant digit first and then least significant digit first?

When adding and subtracting on a number line we start with the most significant digit first (e.g. add the tens then add the units). This is why horizontal expansion starts with the most significant digit first. Once the children are secure in this, it changes to adding the least significant digit first. This bridges the gap between these two stages (many children will only need to see it a few times to understand the relationship but others may need more experience at each stage.

Generalisations

Investigate when re-ordering works as a strategy for subtraction.

Partition the 13 into 10 and 3, 'carry' the ten into the tens column, writing it as 1 to represent one ten." N.B. the '1' can be written at the bottom of the calculation. **It is NOT "carry the 1"**

+ / = signs and missing numbers

Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.

N.B. Please refer to the end of year expectation for the size and range of numbers to be used e.g. ThHTO, decimals, etc.

Generalisation

Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9.

What do you notice about the differences between consecutive square numbers?

Investigate a – b = (a-1) – (b-1) represented visually.

Some Key Questions

What do you notice?

What's the same? What's different?

Can you convince me?

How do you know?

decimals with up to 3 places

13.86 + 9.481 = 23.341

```
  13.86
+  9.481
  23.341
  1 1 1
```

Expectations in Yr 5, 6

12350 + 4921

```
  12350
+  4921
  17271
   1
```

Expectations in Yr 5,6

Revert to expanded methods if the children experience any difficulty.

+ / = signs and missing numbers

Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.

N.B. Please refer to the end of year expectation for the size and range of numbers to be used e.g. ThHTO, decimals, etc.

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.

Sometimes, always or never true? Subtracting numbers makes them smaller.

Some Key Questions

What do you notice?

SUBTRACTION GUIDELINES – Key language which should be used: take away, less than, the difference, subtract, minus, fewer, decrease, '7 take away 3, the difference is		
Stage One	**Stage Two**	**Stage Three**
What do you notice? What is same? What's different? **Prerequisite skills** (based on the practical)	There are two concepts linked to subtraction:	How do you know? **Use known number facts and place value**
Can you convince me? How do you know?		

End of Year Objectives for Addition

Yr1 – recall and jottings for O+O, T+O, T+T, TO+O (within 20 including 0)
Yr2 – TO+O, T+TO, TO+TO, O+O+O
Yr3 – mental methods for HTO + O, HTO+T, HTO+H; written methods for HTO+TO, HTO+HTO
Yr4 – written methods as above and ThHTO+ThHTO, O.t+O.t, £O.th+£O.th
Yr5 – written method for addition of numbers with more than four digits; 2 or more integers, decimals with 2dp e.g. 29.78 + 54.34
Yr6 – As above using increasingly larger numbers

Differentiation Steps for each Stage:

- Not crossing tens
- Crossing Tens
- Crossing Hundreds Only
- Crossing Tens and Hundreds

In addition:

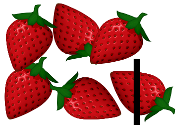
- The number line must be modelled as an image to support calculation from Reception to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- If the calculation can be carried out mentally then do not give it to practice vertical calculation, e.g. TO + TO should not be calculated vertically.

Always present calculations horizontally in order to consider mental calculations.

Number bonds to 10.

Counting back from 20. Find one less than a given number.

Subtract using quantities and objects 2 single digit numbers.



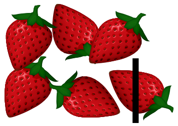
Count back to subtract single digit numbers.

There are two concepts linked to subtraction:

Subtract – where it is natural to count back to 'take away'.

Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].

Understand subtraction as 'take away'.



Concrete –

Physically taking away and removing objects from a whole *(use various objects too). Rather than crossing out, children will physically remove*

Subtract – where it is natural to count back to 'take away'.
Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].

Making 10 *(using Numicon or ten frames)*

14 – 5

Children could also do this by subtracting a 5 from the 10.

Use known number facts and place value to subtract

Using knowledge of number bonds to subtract mentally from multiples of 10s e.g. 30 – 4

Expectations in Yr 2,3

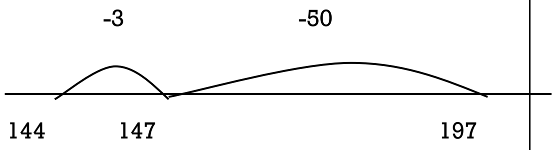
Using knowledge of number bonds to mentally subtract multiples of 10 from multiples of 10 e.g. if 7 – 4 = 3 then 70 – 40 = 30

Using knowledge of number bonds to subtract mentally e.g. if 8 – 3 = 5 then 28 – 3 = 25

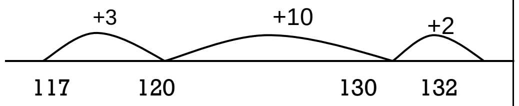
Use of T10 for TO–O (Target 10)
22 – 5 = 22 – 2
= 20 – 3

to subtract

Continue as in Stage 2 but with appropriate numbers e.g.197 – 53 = 144



Use counting on to find the difference
132 – 117 = 15



Secure knowledge in use of J10 and T10 to count back and find the difference.

TO–TO, HTO–TO, HTO–HTO

***By the end of this stage children should know complements to 100. They can then use this knowledge to calculate* HTO–TO, HTO–HTO.**

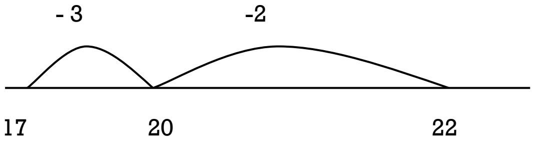
Expectations in Yr 3

Subtract *mentally* a 'near multiple of 10' to or from a two-digit number

Continue as in Stage 2 but with appropriate numbers e.g. 78 – 49 is the same as 78 – 50 +

the objects.

4 – 3 = 1



1

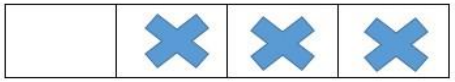
- = signs and missing numbers (inverse)

Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.

Pictorial –

Children to draw the concrete resources they are using and cross out.

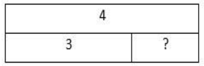
Use of the bar model:

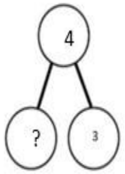


Abstract –

4- 3 =

□ ***= 4 - 3***





Expectations in Yr 1

Use practical and informal written methods to support the subtraction of a one-digit number from a one-digit or two-digit number and a multiple of 10 from a two-digit number.

Concrete –

Counting back *(using number lines or number*

Use of J10 using multiples of 10

Example: 80 – 30

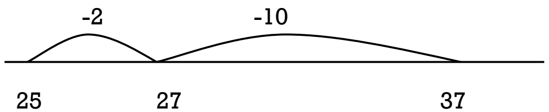
Use of J10

37 – 12 = 37 – 10 – 2

= 27 – 2

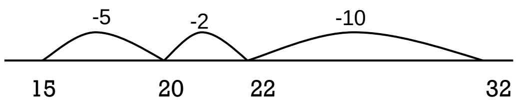
= 25

Expectations in Yr 2,3

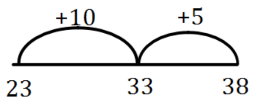


Use of T10 where necessary

32 - 17



Subtraction for finding the difference using counting on e.g. 38 – 23



Pictorial –

14 – 5 = 9

Children to present the ten frame pictorially.

Generalisations

Noticing what happens to the digits when you count in tens and hundreds.

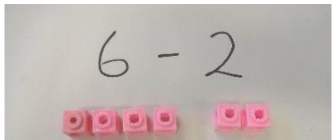
Odd – odd = even etc

Inverses and related facts – develop fluency in finding related addition and subtraction facts.

Develop the knowledge that the inverse relationship can be used as a checking method.

Key Questions

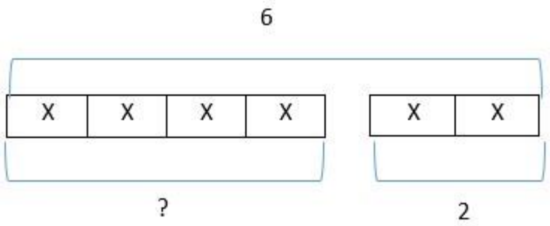
What do you notice? What patterns can you see?



tracks).

Pictorial –

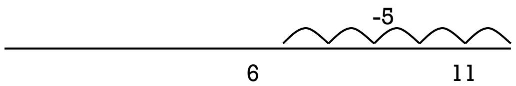
Children to represent what they see pictorially, e.g.



Abstract –

I have 11 toy cars. I lost 5 of them. How many are left?

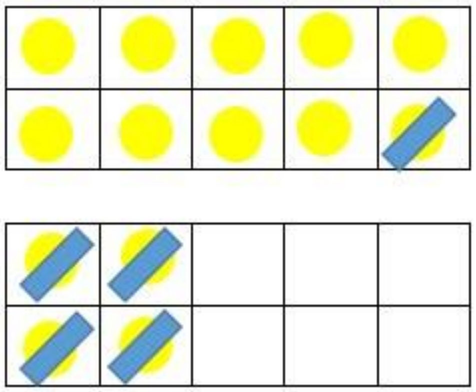
Start with bead strings / bars and move onto number lines below.



Use the vocabulary related to subtraction and symbols to describe and record subtraction number sentences (for the example above it would be 11 – 5 = 6)
Recording by - drawing jumps on prepared lines / tracks.

Use practical resources to find the difference between two small numbers (e.g. 6 and 7).

Count on from smallest to largest number to find the difference where numbers are close in value (e.g. 9 – 7).

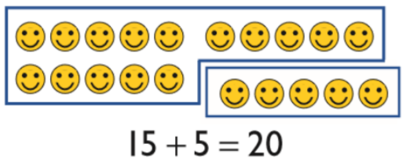


- = signs and missing numbers(inverse)
Continue using a range of equations as in Stage 1 but with appropriate numbers.

Extend to 14 + 5 = 20 – □ (inverse)

Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd – odd = even; odd – even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.

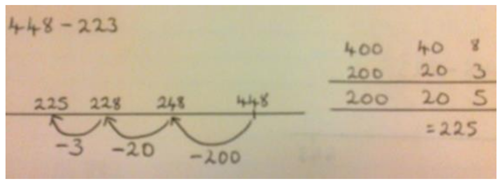


When comparing two methods alongside each other:

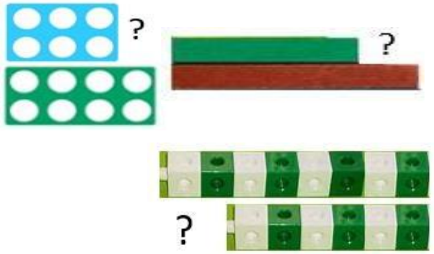
What's the same?

What's different?

Look at this number in the formal method; can you see where it is in the expanded method / on the number line



Finding the difference *(using cubes, Numicon or Cuisenaire rods, other objects can also be used).*



- = signs and missing numbers(inverse)

7 - 3 = □	□ = 7 - 3
7 - □ = 4	4 = □ - 3
□ - 3 = 4	4 = 7 - □
□ - ∇ = 4	4 = □ - ∇

Expectations in Yr 1, 2

Some Key Questions

How many more to make…? How many more is… than…? How much more is…? How many are left/left over? How many have gone? One less, two less, ten less… How many fewer is… than…? How much less is…?

What can you see here?

Is this true or false?

Some Key Questions

How many more to make…?
How many more is… than…?
How much more is…?
How many are left/left over?
How many fewer is… than…?
How much less is…?

Is this true or false?

If I know that 7 + 2 = 9, what else do I know? (e.g. 2 + 7 = 9; 9 – 7 = 2; 9 – 2 = 7; 90 – 20 = 70 etc).
What do you notice? What patterns can you see?

SUBTRACTION GUIDELINES

(- = signs and missing numbers: Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.)

Stage Four	Stage Five	Stage Six

SUBTRACTION GUIDELINES

(- = signs and missing numbers: Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.)

Stage Four

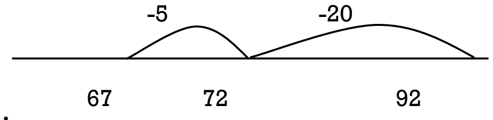
<u>Find a small difference by counting up (relating to inverse)</u>

e.g. 5003 – 4996 = 7

This can be modelled on an empty number line (see complementary addition). Children should be encouraged to use known number facts to reduce the number of steps.

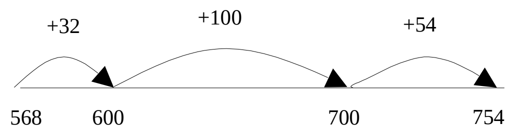
<u>Use known number facts and place value to subtract</u>

92 – 25 = 67



<u>Counting on</u>

Use of number facts to count up to find the difference (T10, T100). 754 – 568 = 186



For those children with a secure mental image of the number line they could record the jumps only:

754 – 568 = 186

Expectations in Yr 3, 4

```
  754
- 568
   32 (600)
  100 (700)
   54 (754)
  186
```

<u>Use known number facts and place value to subtract</u>

Expectations in Yr 4,5

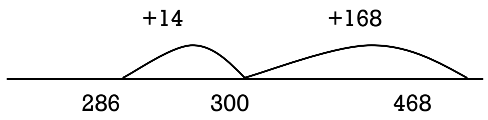
Stage Five

<u>Counting on</u>

Use of number facts to count up to find the difference (T10, T100). This is used in the context of inverse.

14 + 168 = 182 so:

468 – 286 = 182



OR

754 – 286 = 468

Expectations in Yr 3

```
  754
- 286
   14 (300)
  454 (754)
  468
```

Reduce the number of steps to make the calculation more efficient.

Extend to 2 places of decimals.

SUBTRACTION BY EXPANDED DECOMPOSITION (With higher attainers secure in number facts and use of the number line).

Subtracting with no repartitioning needed:

345 – 123

Expectations in Yr 3

```
    3 0 0  +  4 0  +  5
 -  (1 0 0  +  2 0  +  3)
 ------------------------
    2 0 0  +  2 0  +  2
```

Partitioning each number and working from right to left, subtracting the bottom number form the top.

Stage Six

<u>Progress to 4 digit numbers</u>

Teach on a number line first to subtract using T10, T100, T1000 (children should choose the most efficient method) either counting on or counting back.

e.g. 8000 – 2785 = 5215

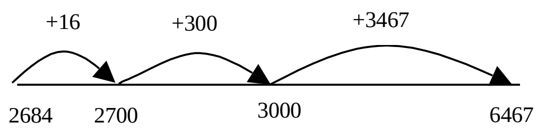
To make this method more efficient, the number of jumps should be reduced to a minimum through children knowing:

- Complements to 1, involving decimals to two decimal places (0.16 + 0.84).
- Complements to 10, 100 and 1000.

<u>Counting on</u>

Expectations in Yr 4

6467 – 2684 = 3783



OR

6467 – 2684 = 3783

```
  16 (2700)     can be refined to     316 (3000)
 300 (3000)                          3467 (6467)
3467 (6467)                          3783
3783
```

Reduce the number of steps to make the calculation more efficient.

Extend to 2 places of decimals.

<u>Concrete:</u>

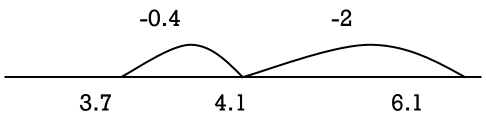
Column method *(using base 10 and having to exchange)* ***45-26***

SUBTRACTION GUIDELINES

(- = signs and missing numbers: Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.)

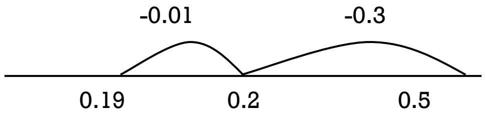
Stage Four

6.1 – 2.4 = 3.7



Use known number facts and place value to subtract

0.5 – 0.31 = 0.19



N.B. Please refer to the end of year expectation for the size and range of numbers to be used e.g. ThHTO, decimals, etc.

Generalisations

Investigate when re-ordering works as a strategy for subtraction.

Eg. 20 – 3 – 10 = 20 – 10 – 3,

but 3 – 20 – 10 would give a different answer.

Some Key Questions

What do you notice?

Stage Five

Express each part as its value represented, i.e. "40 – 20".

Moving on to subtracting with repartitioning of tens only:

2 52 – 114

```
  200  +  50  +  2
- (100 +  10  +  4)
--------------------
                 ?
```

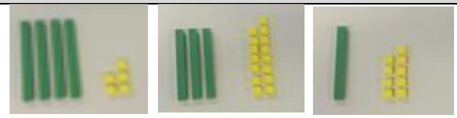
```
         40       12
  200  + ~~50~~ + ~~2~~
- (100 +  10  +   4)
--------------------
  100  +  30  +   8
```

Again, partitioning each number and working from right to left, subtracting the bottom number from the top. Where the subtraction is not possible i.e. 2 – 4, the next value is "REPARTITIONED". So, "repartition 50 + 2 into 40 + 12". It is important to cross out the whole number and replace completely. Do NOT put 'a one in the air'! (It is not a 1, it is a 10.) Then repeat the subtraction process, this time "12 – 4 = 8" and "40 – 10 =30"

N.B. Please refer to the end of year expectation for the size and range of numbers to be used e.g. ThHTO, decimals, etc.

Generalisation

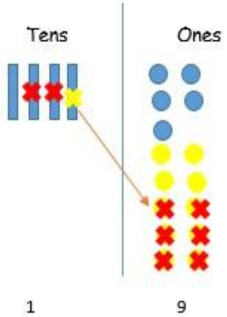
Stage Six



1) *Start by partitioning 45*
2) *Exchange one ten for ten more ones.*
3) *Subtract the ones, then the tens.*

Pictorial –

Represent the base 10 pictorially



Subtraction by Standard Decomposition

346 – 128

$$
\begin{array}{r}
3\ {}^{3}\cancel{4}\ {}^{1}6 \\
-\ 1\ \ 2\ \ 8 \\
\hline
2\ \ 1\ \ 8
\end{array}
$$

It is still vital that the correct language of place value is used. The tens are REPARTITIONED (not "'borrow' a 1" and it is not "3 takeaway 1" but "300 take away/ subtract/minus 100").

N.B. Please refer to the end of year expectation for the size and range of numbers to be used e.g. ThHTO, decimals, etc.

Generalisations

SUBTRACTION GUIDELINES (- = signs and missing numbers: Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.)		
Stage Four	**Stage Five**	**Stage Six**
What's the same? What's different? *Can you convince me?* *How do you know?*	Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9. What do you notice about the differences between consecutive square numbers? Investigate a – b = (a-1) – (b-1) represented visually. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

End of Year Objectives for Subtraction

Year 1 – mentally subtract O–O, TO–O, TO–TO (up to 20 e.g. 15 – 12).
Year 2 – mental and written- TO–O, TO–multiple of 10, TO–TO(mentally with informal jottings)
Year 3 – subtract mentally, HTO–O, HTO–T, HTO–H, TO–O and TO–TO. Formal written methods for TO–TO, HTO–TO, HTO–HTO.
Year 4 – as above and efficient written methods for ThHTO–ThHTO, ThHTO–HTO, O.t–O.t, £O.th–£O.th.
Year 5 – Efficient written methods for subtraction of 2 integers with more than 4 digits e.g. 45230–12432 and decimals with up to 2dp e.g. 54.34–29.78.
Year 6 – as above with increasingly larger numbers

Please note:
There are two concepts linked to subtraction:
Subtract - where it is natural to count back to 'take away'.
Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].

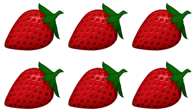
- Children should not move on to a written method if they are not completely confident with using a number line.
- Children will need to have had experience of different types of jumping on a number line e.g. T10 (target the ten), J10 (jump in 10s) and know how to partition numbers in different ways.
- These methods can also be easily applied, at different levels, to finding differences in values of money, measures and time.

Always present calculations horizontally in order to consider mental calculations first.

MULTIPLICATION GUIDELINES

Stage One

Prerequisite skills
Multiplication is related to known facts including doubling and counting groups of the same size.



3 + 3

E.g. use of dominoes and dice.

Counting using a variety of practical resources

Numicon and bead strings

Counting in 2s e.g. counting socks, shoes, animal's legs…

Counting in 5s e.g. counting fingers, fingers in gloves, toes…

Counting in 10s e.g. fingers, toes…

Pictures / marks

There are 2 socks in a pair How many socks are there in 3 pairs?

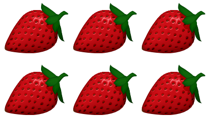
Concrete:

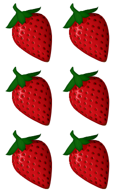
Repeated grouping/repeated addition

(does not have to be restricted to cubes)

Stage Two

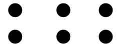
Arrays and repeated addition



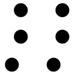


Looking at rows
3 + 3
2 groups of 3

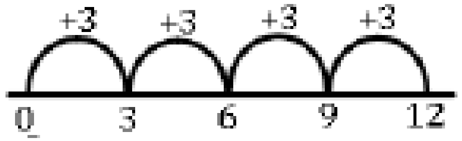
Looking at rows
2 + 2 + 2
3 groups of 2



3 x 2 or 3 + 3



2 +2 + 2 or 2 x 3



If the calculation is 3 x 4 for example, children should understand that this means 3 + 3 + 3 + 3. Children should also understand the commutative law and be able to use 4 x 3.

Expectations in Yr 1, 2

Concrete:

Use number lines to show repeated groups- e.g- 3 x 4

Stage Three

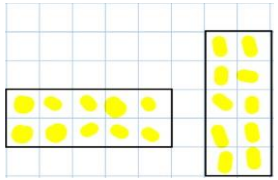
Concrete:

Use arrays to illustrate commutativity *(counters and other objects can also be used)*

2 x 5 = 5 x 2

Pictorial:

Children to draw the arrays



Abstract:
Children to be able to use an array to write a range of calculations e.g.
2 x 5 = 10

5 x 2 = 10

2 + 2 + 2 + 2 + 2 = 10 and 5 + 5 =10

Arrays and repeated addition
Continue to understand multiplication as repeated addition and continue to use arrays and number lines (as in Stage 2).

Use known facts and place value to carry out simple multiplications through

Partition
23 x 3 =

3 x 4 or 3 lots of 4

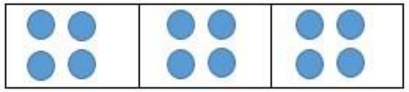
The above is required before moving on to Stage 2

Pictorial:

Children to represent the practical resources in a picture e.g.

XX XX XX
XX XX XX

Use of a bar model for a more structured method



Abstract :

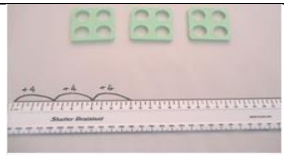
3 x 4 = 12

IS SAME AS 4 + 4 + 4

Generalisations

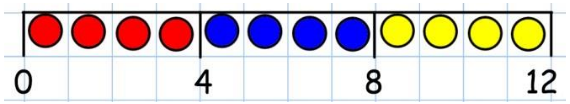
Understand 6 counters can be arranged as 3+3 or 2+2+2

Understand that when counting in twos, the numbers



Pictorial:

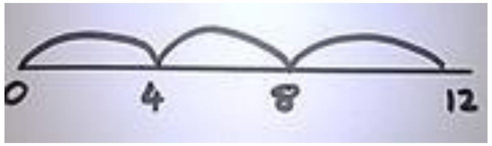
Represent this pictorially alongside a number line e.g:



Abstract:

Abstract number line

3 x 4 = 12



x = signs and missing numbers

7 x 2 = □	□ = 2 x 7
7 x □ = 14	14 = □ x 7
□ x 2 = 14	14 = 2 x □
□ x ∇ = 14	14 = □ x ∇

Expectations in Yr 1, 2

Partitioning

Children need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways: 6 = 5 + 1 so

e.g. Double 6 is the same as double five add double one.

Generalisation

Commutative law shown on array

X	20	3
3	3 x 20 = <u>60</u>	3 x 3 = <u>9</u>

OR

2 | 3 X 3 = 69

2 | 0 X 3 = 60

Expectations in Yr 3

3 x 3 =9

Moving on to: multiplying by 10 (EMPHASISE on the lay out)

x	10	6
10	10 x 10= <u>100</u>	10 x 6 = <u>60</u>
7	7 x 10 = <u>70</u>	7 x 6 = <u>42</u>

100 + 60 + 70 + 42 = 272

At the end of Stage 3 the children should know their 12 x 12 times tables.

Generalisations

Connecting x2, x4 and x8 through multiplication facts

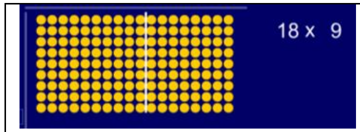
Comparing times tables with the same times tables which is ten times bigger. If 4 x 3 = 12, then we know 4 x 30 = 120. Use place value counters to demonstrate this.

are always even. **Some Key Questions** Why is an even number an even number? What do you notice? What's the same? What's different? Can you convince me? How do you know?	Repeated addition can be shown mentally on a number line Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?) **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

MULTIPLICATION GUIDELINES- *Key language which should be used: double times, multiplied by, the product of, groups of, lots of, 'is equal to' 'is the same as'*

Stage Four	Stage Five	Stage Six
x = signs and missing numbers Continue using a range of equations as in Stage 3 but with appropriate numbers **Partition** Continue to use arrays:	**Partition** 47 x 6 = 282 47 x 6 = (40 x 6) + (7 x 6) = 282 OR Use the grid method of multiplication (as below) Grid method	**Grid method** 372 x 24 is approximately 400 x 20 = 8000 Extend to decimals with up to two decimal places. The recording is reduced further, with carry digits recorded below the line.

Expectations in Yr 5, 6



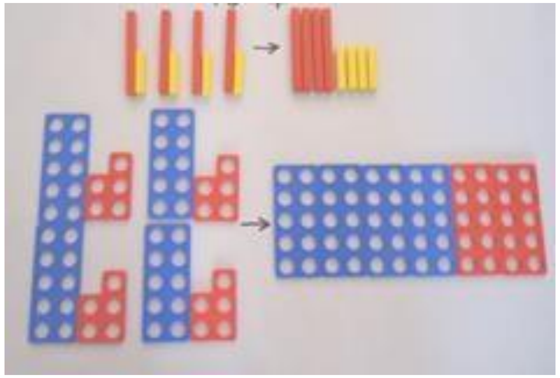
18 x 9 = 162

18 x 9 = (10 x 9) + (8 x 9) = 162

Use Multiplication array ITP to model partitioning into tens and ones, using the familiar visual pattern of 5s.

Concrete: Partition to multiply (use numicon, base 10, Cuisenaire rods)

4 x 15



Abstract: Children to be encouraged to show the steps they have taken

4 x 15

10 5

10 x 4 = 40
5 x 4 = 20
40 + 20 = 60

72 x 38 is approximately 70 x 40 = 2800

Remember, always present calculations horizontally in order to consider mental calculations first.

Again, if the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23 x 15 should not be calculated vertically. Consider use of numbers carefully. Avoid numbers which involve x 2, x 4, x 5, x 8 which can be solved mentally using known facts.

Expectations in Yr 4, 5

382 x 23 =

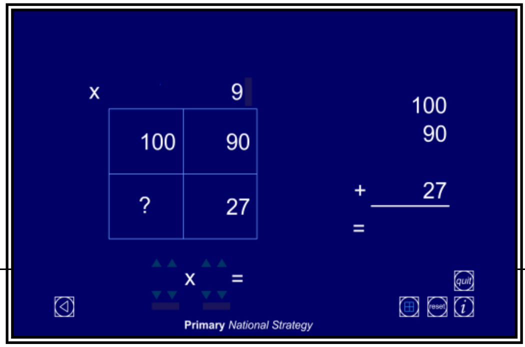
x	**300**	**80**	**2**
20	20 x 300 = 6000	20 x 80 = 1600	20 x 2 = 40
3	3 x 300= 900	3 x 80 = 240	3 x 2 = 6

6000 + 1600 + 900 + 240 + 240 + 40 + 6 = 8986

6000 + 2500 + 480 + 46 = 8000+ 980 +46

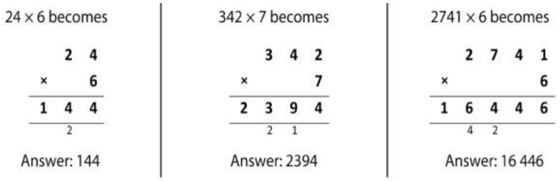
It is important to write the calculation in the grid for both the pupil and teacher to be able to identify errors made in multiplication facts or in the calculating the process.

Use Multiplication grid ITP to assess understanding and application of the grid method by 'hiding' the question parts and 'revealing' some of the answer parts.

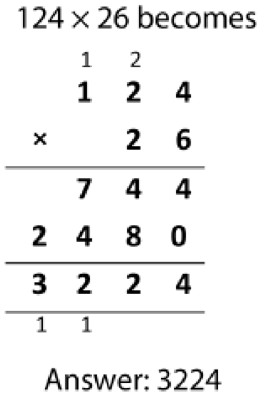


38
x 7
266
5

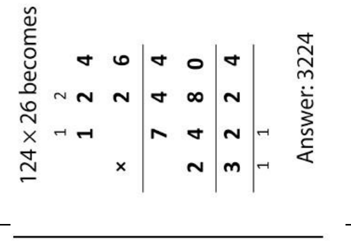
Short multiplication:

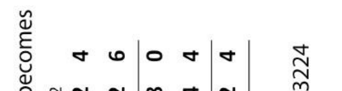


Long multiplication

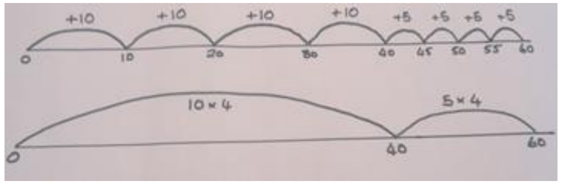


Note- multiplying with Tens first or by Ones





A number line can also be used



Use the grid method of multiplication (as below)

36 x 27 =

x	30	6
20	20 x 30= **600**	20 x 6 = **120**
7	7 x 30 = **210**	7 x 6 = **42**

Expectations in Yr 3,4

Generalisations

Children given the opportunity to investigate numbers multiplied by 1 and 0.

	6	0	0
	1	2	0
+	2	1	0
		4	2
	9	7	2

When they know multiplication facts up to x12, do they know

You can extend to using the grid method to multiply decimals

Expanded Column Multiplication

Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 382 × 23 is 'three hundreds multiplied by twenty', not 'three times two', although the relationship 3 × 2 should be stressed.

Expectations in Yr 4, 5

Most significant first

382 x 23 =

300 + 80 + 2
X 20 + 3
6000 (20 x 300)
1600 (20 x 80)
40 (20 x 2)
900 (3 x 300)
240 (3 x 80)
6 (3 x 2)
8786

300 + 80 + 2
X 20 + 3
6000
1600
40
900
240
6
8786

Least significant first

382 x 23 =

300 + 80 + 2
X 20 + 3
6 (3 x 2)
240 (3 x 80)
900 (3 x 300)
40 (20 x 2)
1600 (20 x 80)
6000 (20 x 300)
8786

300 + 80 + 2
X 20 + 3
6
240
900
40
1600
6000
8786

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.

Understanding the use of multiplication to support conversions between units of measurement.

what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?) **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	**Generalisation** Relating arrays to an understanding of square numbers and making cubes to show cube numbers. Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000) **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know? How do you know this is a prime number?	**Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

End of Year Objectives for Multiplication

Year 1 – make connections between arrays, number patterns, and counting including practical problems that combine groups of 2, 5 or 10

Year 2 - represent multiplication as repeated + and arrays. Practical and informal written methods and vocabulary used to support multiplication alongside known facts and mental strategies. Understand and use '3 for free' for x and ÷ of the 2, 5 and 10 times-tables.

Year 3 – as above and, recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Describe the effect of Ox10, TOx10, Ox100, TO x 100. Practical and informal written methods for TO x O.

Year 4 – Derive and recall x and ÷ facts up to 12 x 12 and '3 for free' facts. Multiply numbers to 1000 by 10 and 100. Formal written layout and explain TO/HTO x O.

Year 5 – mentally multiply TO x O. Multiply whole numbers and decimals by 10, 100 and 1000. Formal written methods up to multiply ThHTO x O, ThHTO x TO, ThHTO ÷ O, O.t x O

Year 6 – mentally calculate TO x O, O.t X O. Formal written methods to multiply up to 4 digit by 2 digit and one digit with up to 2 decimal places, Formal written methods to divide up to 4 digit by 2 digit

DIVISION GUIDELINES - Key language which should be used: share, group, divide, divided by, half, 'is equal to' 'is the same as'		
Stage One	Stage Two	Stage Three

<u>As with addition and subtraction, before progressing through the stages of calculation:</u>

Learning

- It is crucial to know or be able to derive key number facts:

⇨ Understand and use doubling and halving

⇨ ×/÷ 10 (as moving a place to the left/right <u>NOT</u> "add a zero" etc.!!)

- Place value and partitioning MUST be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching

- The number line and the use of arrays must be modelled as images to support calculation from Reception to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23 x 15 should not be calculated vertically. Consider use of numbers carefully.

Always present calculations horizontally in order to consider mental calculations first.

Prerequisite skills (based on the practical)
Understanding the language of half in different contexts.
Know halves of even numbers up to 10.

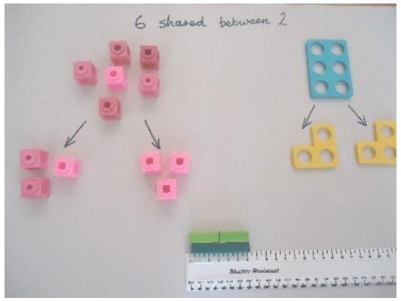
Sharing
Requires secure counting skills
-see counting and understanding number strand

Develops importance of one-to-one correspondence
See appendix for additional information on x and ÷ and aspects of number

Concrete:

Sharing – 6 sweets are shared between 2 people. How many do they have each?

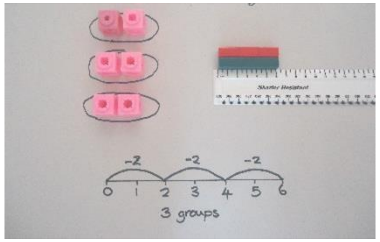
6 shared between 2 *(other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates)*



Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops etc.

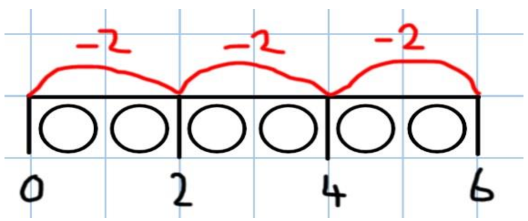
Concrete: ***Understand division as repeated grouping and subtracting***

6 ÷ 2

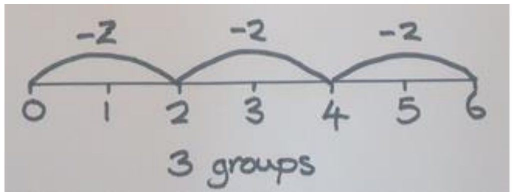


Expectations in Yr 1, 2

Pictorial:



Abstract:



Grouping
Link to counting and understanding number strand
Count up to 100 objects by grouping them and counting in

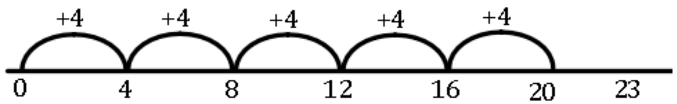
÷ = signs and missing numbers
Continue using a range of equations as in Stage 2 but with appropriate numbers.

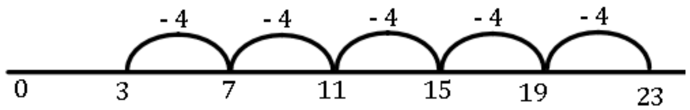
Understand division as sharing and grouping
24 ÷ 3 can be modelled as:
Sharing – 24 shared between 3

OR
Grouping - How many 3's make 24?

Expectations in Yr 2,3

Remainders
23 ÷ 4 = 5r3
Sharing - 23 shared between 4, how many left over?
Grouping – How many 4's make 23, how many left over?
e.g.

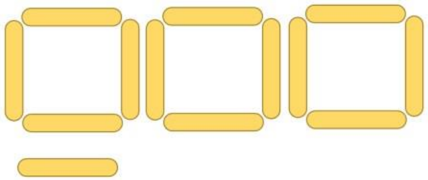




Concrete: (for remainders)

13 ÷ 4 = 3 remainder 1

Use of lollipop sticks to form wholes

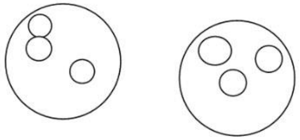


Grouping
Sorting objects into 2s / 5s/ 10s etc.
How many pairs of socks are there?

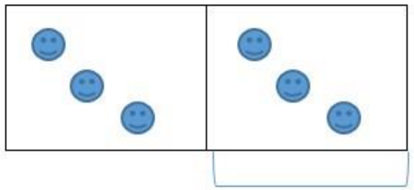
There are 10 bulbs. Plant 5 in each pot. How many pots are there?

Jo has 10 Lego wheels. How many bicycles can she make?

Pictorial:



This can also be done in a bar so all 4 operations have a similar structure:



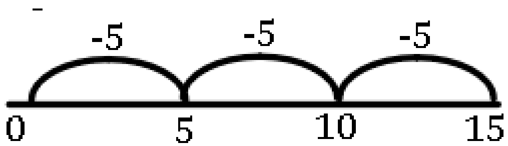
Generalisations

- True or false? I can only halve even numbers.
- Grouping and sharing are different types of

tens, fives or twos;…

Find one half, one quarter and three quarters of shapes and sets of objects
15 ÷ 5 can be modelled as:
There are 15 strawberries.
How many people can have 5 each? How many 5s make 15?

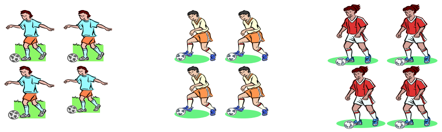
15 ÷ 5 can be modelled as repeated subtraction



In the context of money count forwards and backwards using 2p, 5p and 10p coins

Practical grouping e.g. in PE

12 children get into teams of 4 to play a game. How many teams are there?



Children should know that division is not commutative.

÷ = signs and missing numbers

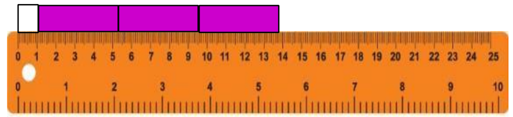
6 ÷ 2 = □	□ = 6 ÷ 2
6 ÷ □ = 3	3 = 6 ÷ □
□ ÷ 2 = 3	3 = □ ÷ 2
□ ÷ ∇ = 3	3 = □ ÷ ∇

Expectations in Yr 1, 2.

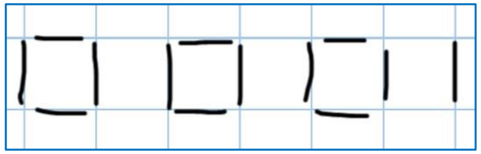
Generalisations

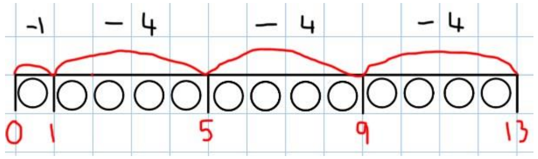
Noticing how counting in multiples if 2, 5 and 10 relates

Use of Cuisenaire rods and rulers (using repeated subtraction)

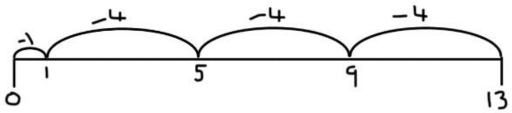


Pictorial:





Abstract:



Generalisations

Inverses and related facts – develop fluency in finding related multiplication and division facts.

Develop the knowledge that the inverse relationship

problems. Some problems need solving by grouping and some by sharing. Encourage children to practically work out which they are doing. **<u>Some Key Questions</u>** How many groups of…? How many in each group? Share… equally into… What can do you notice?	to the number of groups you have counted (introducing times tables) An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?) Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group. **<u>Some Key Questions</u>** How many 10s can you subtract from 60? I think of a number and double it. My answer is 8. What was my number? If 12 x 2 = 24, what is 24 ÷ 2? Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?)	can be used as a checking method. **<u>Some Key Questions</u>** Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?) What is the missing number? 17 = 5 x 3 + __ __ = 2 x 8 + 1

DIVISION GUIDELINES - Key language which should be used: share, group, divide, divided by, half, 'is equal to' 'is the same as'

Stage Four

÷ = signs and missing numbers

Continue using a range of equations as in Stage 3 but with appropriate numbers.

Sharing and grouping

60 ÷ 12 can be modelled as:

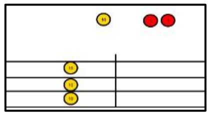
grouping – 12 subtracted repeatedly from 60 on a no. line, leading to subtracting 'groups' of 12.

sharing – sharing among 12, the number given to each person.

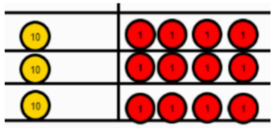
Concrete:

Sharing using place value counters.

42 ÷ 3= 14

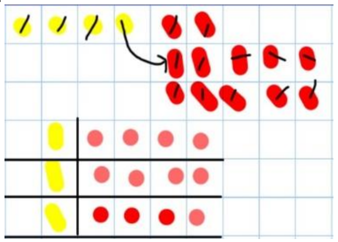


1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10?
Exchange the ten for 10 ones and share out 12 ones



Expectations in Yr 2, 3

Pictorial:

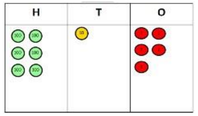


Stage Five

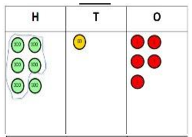
Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

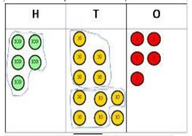
615 ÷ 5



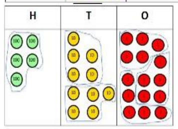
Step 1: make 615



Step 2: Circle your groups of 5



Step 3: Exchange 1H for 10T and circle groups of 5



Step 4: exchange 1T for 10ones and circles groups of 5

Remainders

Pencil and paper procedures- Chunking

256 ÷ 7 lies between 210 ÷ 7 = 30 and 280 ÷ 7 = 40

Key Facts
1 x 7 = 7
2 x 7 = 14
5 x 7 = 35
10 x 7 =

	256		
-	<u>210</u>	7 x 30	
	46		
-	<u>42</u>	7 x 6	
	4		

Answer: 36 remainder 4

Stage Six

Sharing, grouping and remainders as Stage Five

Pencil and paper procedures- Chunking

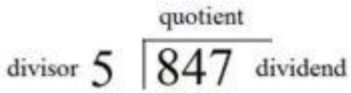
977 ÷ 36 is approximately 1000 ÷ 40
=

Key Facts
1 x 36 = 36
2 x 36 = 72
5 x 36 = 180
10 x 36 = 360

	977	
-	<u>720</u>	36 x 20
	257	
-	<u>180</u>	36 x 5
	77	
-	<u>72</u>	36 x 2
	5	

Answer: $27\ ^{5}/_{36}$

Pencil and Paper procedures- Short Division Method



Expectations in Yr 5, 6

Expectations in Yr 3, 4

ecomes

$$\begin{array}{r|l} & 4\ \ 5\ \ r1 \\ \hline 1\ 1 & 4\ \ 9\ \ {}^{5}6 \end{array}$$

Answer: $45\frac{1}{11}$

Abstract:

42 ÷ 3

42 = 30 + 12

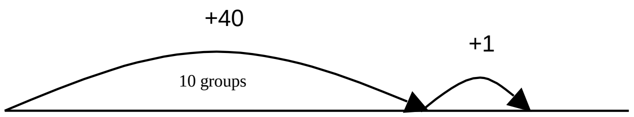
30 ÷ 3 = 10

12 ÷ 3 = 4

10 + 4 = 14

Remainders

41 ÷ 4 = 10 r1



41 = (10 x 4) + 1

Pencil and paper procedures- Chunking.

72 ÷ 5 lies between 5 0 ÷ 5 = 10 and 100 ÷ 5 = 20

Key Facts
1 x 5 = 5
2 x 5 = 10
5 x 5 = 25
10 x 5 = 50

	72	
-	**50**	**(10 groups)**
	22	
-	**20**	**(4 groups)**
	2	

Answer : 14 remainder 2

Quotients expressed as fractions or decimal fractions

61 ÷ 4 = 15 ¼ or 15.25

Also, Short Division for More Able Children

432 ÷ 5 becomes

$$\begin{array}{r|l} & 8\;\;6\;\;r\,2 \\ \hline 5 & 4\;\;{}^{3}3\;\;2 \end{array}$$

Answer: 86 remainder 2

Considering each column starting from the left.

Expectations in Yr 5, 6

Generalisations

The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g.

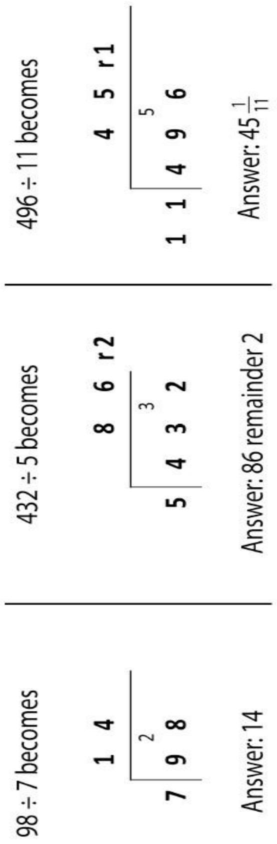
Start: **24 = 24**

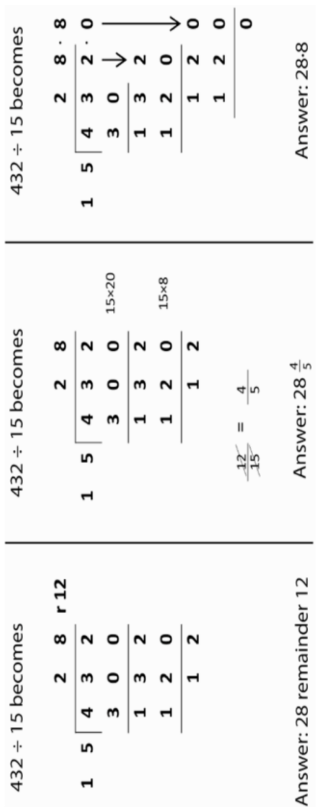
Player 1: **4 x 6 = 24**

Player 2: **4 x 6 = 12 x 2**

Player 1: **48 ÷ 2 = 12 x 2**

Both methods above are necessary at this stage, to deal with the wide range of problems experienced at Stage Six.





/Generalisations

Generalisations

True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?

Is it sometimes, always or never true that □ ÷ Δ = Δ ÷ □?

Inverses and deriving facts. 'Know one, get lots free!' e.g.: 2 x 3 = 6, so 3 x 2 = 6, 6 ÷ 2 = 3, 60 ÷ 20 = 3, 600 ÷ 3 = 200 etc.

Some Key Questions

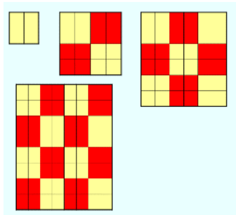
Sometimes, always, never true questions about multiples and divisibility. (When looking at the examples on this page, remember that they **may not** be 'always true'!) E.g.:

- Multiples of 5 end in 0 or 5.
- The digital root of a multiple of 3 will be 3, 6 or 9.
- The sum of z` 4 even numbers is divisible by 4.

Some Key Questions

Sometimes, always, never true questions about multiples and divisibility. E.g.:

- If the last two digits of a number are divisible by 4, the number will be divisible by 4.
- If the digital root of a number is 9, the number will be divisible by 9.



- When you square an even number the result will be divisible by 4 (one example of 'proof' shown left)

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.

Some Key Questions

Sometimes, always, never true questions about multiples and divisibility. E.g.: If a number is divisible by 3 and 4, it will also be divisible by 12.

Using what you know about rules of divisibility, do you think 7919 is a prime number? Explain your answer.

End of Year Objectives for Division

Year 1 – practical problems that share into equal groups of 2, 5 or 10.

Year 2 – derive and recall division facts for 2, 5 or 10, represent division as repeated subtraction (grouping) and sharing. Practical and informal written methods and vocabulary used to support division, including remainders. To know that division is not commutative.

Year 3 – Practical and informal written methods for TO÷O. Understand and use '3 for free' for x and ÷ of the 2, 3, 4, 5, 6, 8 and 10 times-tables. Round remainders up or down, depending on the context.

Year 4 – Derive and recall x facts up to 12x12 and apply '3 for free' facts. Divide numbers to 1000 by 10 and 100. Develop and use formal written layouts to record.

Year 5 – Divide whole numbers and decimals by 10, 100 and 1000. Divide numbers up to 4 digits by a one digit number using the formal written methods for division and interpret remainders appropriately for the context.

Year 6 – Divide numbers up to 4 digits by a 2 digit whole number using the formal written method of long division interpreting remainders as fractions, decimals, etc. Divide numbers up to 4 digits by a two digit number using the formal written methods for division and interpret remainders appropriately for the context.

Learning

- It is crucial to know or be able to derive key number facts:
  - ⇨ Understand and use doubling and halving
  - ⇨ ×/÷ 10 (as moving a place to the left/right NOT "add a zero" etc.!!)
- Place value and partitioning MUST be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching

- The number line and the use of arrays must be modelled as images to support calculation from Reception to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 24 ÷ 3 should not be calculated using short division. Consider use of numbers carefully.

Always present calculations horizontally in order to consider mental calculations first.